JEE(Advanced) EXAMINATION – 2023

(Held On Sunday 04th June, 2023)

PAPER-2

PHYSICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

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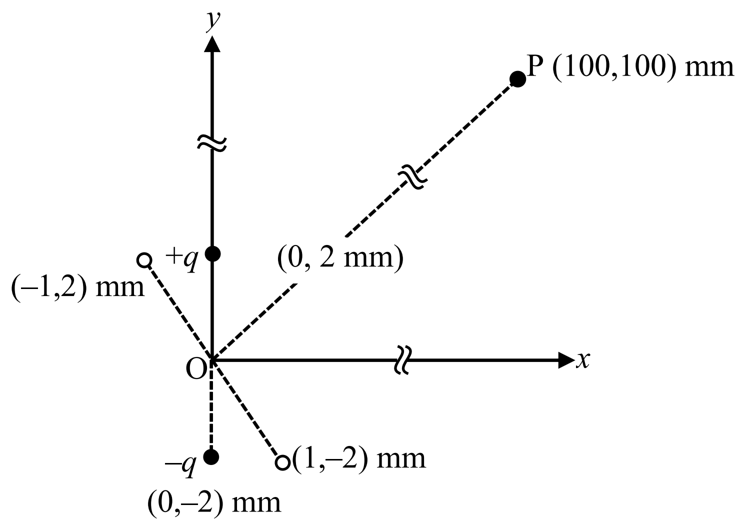
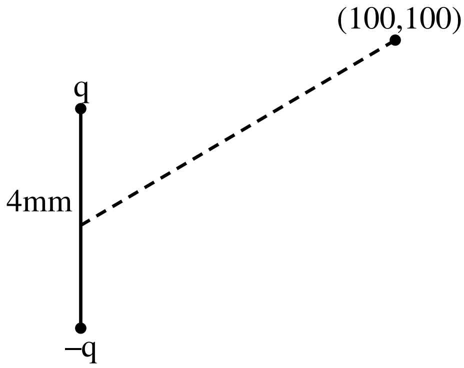
1. An electric dipole is formed by two charges $+q$ and $-q$ located in xy-plane at (0, 2) mm and (0, –2) mm, respectively, as shown in the figure. The electric potential at point P(100, 100) mm due to the dipole is V_0. The charges $+q$ and $-q$ are then moved to the points (–1, 2) mm and (1, –2) mm, respectively. What is the value of electric potential at P due to the new dipole ?



(A) $V_0/4$ (B) $V_0/2$ (C) $V_0/\sqrt{2}$ (D) $3V_0/4$

Ans. (B)

Sol.

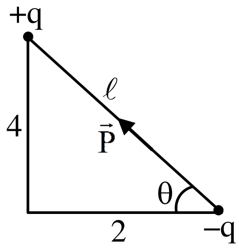


$P_1 = q(4)$

$\vec{P}_1 = P_1\hat{j}$

$\vec{r} = 100(\hat{i} + \hat{j})mm$

$$v_0 = \frac{K P_1 . \vec{r}}{r^3} = \frac{K(100P_1)}{(100\sqrt{2})^3}$$



$\tan\theta = 2$

$\vec{P}_2 = P_2\left[-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$

$\vec{r} = 100(\hat{i} + \hat{j})mm$

$P_2 = q\ell$

$$v = \frac{K\vec{P}_2.\vec{r}}{r^3}$$

$$v = \frac{K(100P_2) - (-\cos\theta + \sin\theta)}{(100\sqrt{2})^3}$$

$$\frac{v_0 P_2}{P_1} = (-\cos\theta + \sin\theta)$$

$$v = v_0 \frac{q\ell}{q(4)}[-\cos\theta + \sin\theta]$$

$$= \frac{v_0}{4}[-2+4] = \frac{v_0}{2}$$

2. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^{\alpha} h^{\beta} G^{\gamma}$. Which of the following is the correct option ?

(A) $\alpha = 7, \beta = -1, \gamma = -2$
(B) $\alpha = -7, \beta = -1, \gamma = -2$
(C) $\alpha = 7, \beta = -1, \gamma = 2$
(D) $\alpha = -7, \beta = 1, \gamma = -2$

Ans. (A)

Sol. $Y = c^{\alpha} h^{\beta} G^{\gamma}$

$$ML^{-1}T^{-2} = (LT^{-1})^{\alpha}(ML^2T^{-1})^{\beta}(M^{-1}L^3T^{-2})^{\gamma}$$

$1 = \beta - \gamma$ ...(1)

$-1 = \alpha + 2\beta + 3\gamma$ ...(2)

$-2 = -\alpha - \beta - 2\gamma$ ...(3)

$-3 = \beta + \gamma$

$1 = \beta - \gamma$

$-2 = 2\beta \quad \Rightarrow \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6 \quad \therefore \alpha = 7$

3. A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$, where α is a non-zero constant. What is the force $\vec{F}$ acting on the particle ?

(A) $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$
(B) $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$
(C) $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$
(D) $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$

Ans. (A)

Sol. $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$

$v_x = \alpha y \qquad v_y = 2\alpha x$

$$\frac{dv_x}{dt} = \alpha\frac{dy}{dt} = 2\alpha^2 x \qquad \frac{dv_y}{dt} = 2\alpha v_x = 2\alpha^2 y$$

$\therefore \vec{F} = m\vec{a} = 2m\alpha^2(x\hat{x} + y\hat{y})$

4. An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas in n. The internal energy of one mole of the gas is U_n and the speed of sound in the gas is v_n. At a fixed temperature and pressure, which of the following is the correct option ?

(A) $v_3 < v_6$ and $U_3 > U_6$
(B) $v_5 > v_3$ and $U_3 > U_5$
(C) $v_5 > v_7$ and $U_5 < U_7$
(D) $v_6 < v_7$ and $U_6 < U_7$

Ans. (C)

Sol. $U = \frac{1}{2}fnrT = \frac{frT}{2}$

$\therefore$ A and B are wrong.

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\left(\frac{2}{f}+1\right)\frac{RT}{M}}$$

$\Rightarrow$ more 'f', less 'v'

$\therefore v_5 > v_7$

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: –2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

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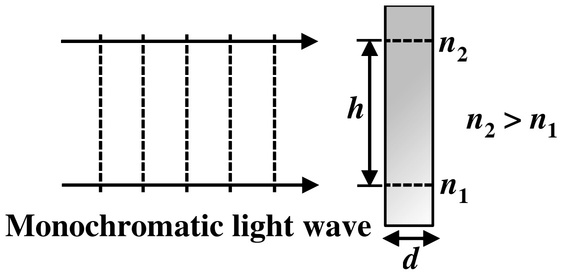
5. A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is (are) true about the light wave emerging out of the slab ?



(A) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2^2 - n_1^2)d}{2h}\right]$

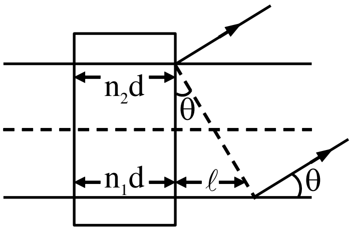
(B) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2 - n_1)d}{h}\right]$

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

Ans. (B, D)

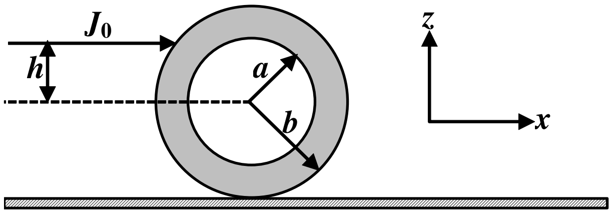
Sol.



$n_1d + \ell = n_2 d$

$$\tan\theta = \frac{\ell}{h} = \frac{(n_2 - n_1)d}{h}$$

6. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct ?



(A) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$

(B) For $\mu \neq 0$ and $a \to b$, $h_m = b$

(C) For $h = h_m$, the initial angular velocity does **not** depend on the inner radius a.

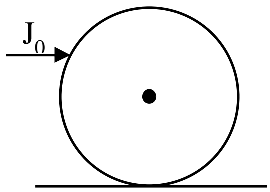
(D) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

Ans. (A,B,C,D)

Sol. $J_0 = mv$ .....(1)

$J_0 h_m = I_c\omega$ ......(2)

$v = \omega R$ ....(3)



$$\Rightarrow h_m = \frac{I_C}{mR}$$

(A) If $a = 0$ $I_c = \frac{1}{2}mb^2$ & $R = b$ $\therefore$ $h_m = \frac{b}{2}$

(B) If $a = b$ $I_C = mb^2$ & $R = b$ $\therefore$ $h_m = b$

(C) $v = \frac{J_0}{m} \Rightarrow 100 = \frac{V}{R} = \frac{J_0}{mR}$

(D) Force is acting on COM $\therefore$ No rotation.

7. The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ V m^{-1}. Which of the following option(s) is(are) correct?

[Given: The speed of light in vacuum, $c = 3 \times 10^8\ ms^{-1}$]

(A) $B_x = -2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm^{-2}.

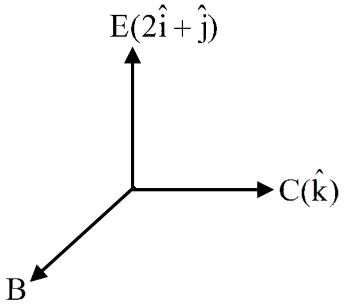
(B) $B_y = 2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm^{-2}

(C) The wave is polarized in the *xy*-plane with polarization angle 30° with respect to the *x*-axis.

(D) The refractive index of the medium is 2.

Ans. (A, D)

Sol.

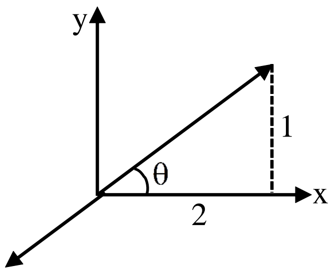


$$C_{medium} = \frac{5\times10^{14}}{10^7/3} = 1.5\times10^8 \text{ m/s} \quad \therefore \mu = 2$$

$$C_{medium} = \frac{E}{B} \Rightarrow B = \frac{E}{C_m} = \frac{30\sqrt{5}}{1.5\times10^8} = 2\sqrt{5}\times10^{-7}$$

$$\vec{B}_{direction} \equiv \hat{k}\times(2\hat{i}+\hat{j}) \equiv \frac{2\hat{j}-\hat{i}}{\sqrt{5}}$$

$$\therefore\ \vec{B} = 2\times10^{-7}(-\hat{i}+2\hat{j})\sin\left[27\left(5\times10^{17}t - \frac{10^7}{3}z\right)\right]$$



$$\tan\theta = \frac{1}{2}$$

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;
Zero Marks : 0 In all other cases

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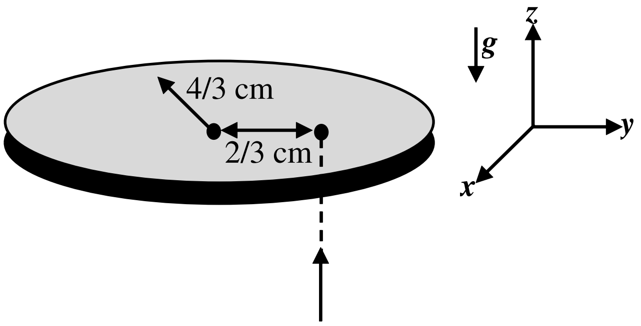
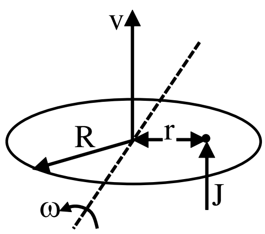
8. A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal *xy*-plane. The coin is tossed vertically up (+*z* direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N-s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the +*z* direction. By the time the coin reaches back to its initial position, it completes *n* rotations. The value of *n* is ____.
[Given: The acceleration due to gravity g = 10 ms^{-2}]



Ans. (30)

Sol.



$J = mv$ ......(1)

$Jr = I_c\omega$ ....(2)

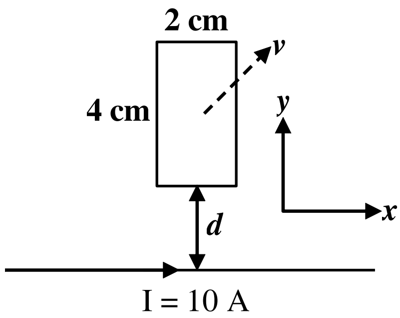
$J_C = \frac{1}{4}mR^2$ ....(3)

$t = \frac{2v}{g}$ ....(4)

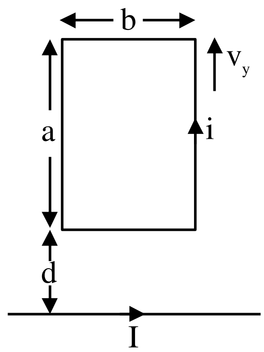
$\theta = 2\pi N = \omega t$ ....(5) $\therefore N = 30$

9. A rectangular conducting loop of length 4 cm and width 2 cm is in the *xy*-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{\sqrt{3}}{2}\hat{x}+\frac{1}{2}\hat{y}$ with a constant speed *v*. The wire is carrying a steady current I = 10 A in the positive *x*-direction. A current of 10 μA flows through the loop when it is at a distance *d* = 4 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of *v* is __________ ms^{-1}.

[Given: The permeability of free space $\mu_0 = 4\pi \times 10^{-7}\,NA^{-2}$]



Ans. (4)



Sol. $R = 0.1\Omega$

$\varepsilon = (B_1 - B_2)bv_y$

$$i = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi R}\left(\frac{1}{d} - \frac{1}{d+a}\right)bv_y$$

$$\Rightarrow 10^{-5} = \frac{2\times10^{-7}\times10}{0.1}\left[\frac{1}{4}-\frac{1}{8}\right]\times 2.v_y$$

$\therefore\ v_y = 2$

$$\tan\theta = \frac{v_y}{v_x} = \frac{1}{\sqrt{3}}$$

θ

$\therefore\ v_x = 2\sqrt{3}$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = 4$$

10. A string of length 1 m and mass 2×10^{-5} kg is under tension T. when the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is _______ Newton.

Ans. (5)

Sol. $f = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

$$750 = \frac{P}{2}\sqrt{\frac{T}{\mu}} \quad ......(1)$$

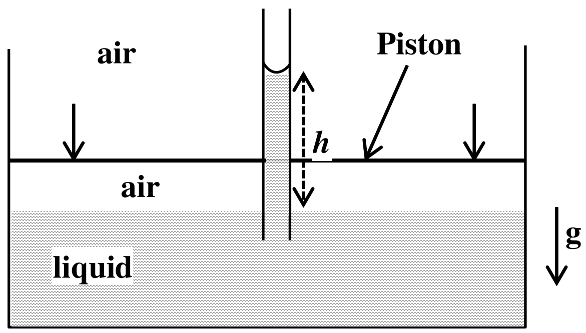
$$1000 = \frac{P+1}{2}\sqrt{\frac{T}{\mu}} \quad .....(2)$$

$$\frac{4}{3} = \frac{P+1}{P} \quad \therefore P = 3$$

$$\Rightarrow 1000 = \frac{4}{2}\sqrt{\frac{T}{2\times10^{-5}}} \quad \therefore T = 5N$$

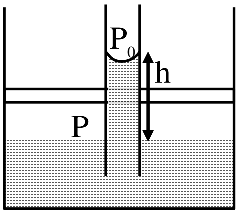
11. An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume V_0 to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (h) of the liquid column in the capillary above the liquid level in cm is_______.

[Given: Surface tension of the liquid is 0.075 Nm^{-1}, atmospheric pressure is 10^5 N m^{-2}, acceleration due to gravity (g) is 10 m s^{-2}, density of the liquid is 10^3 kg m^{-3} and contact angle of capillary surface with the liquid is zero]



Ans. (25)

Sol.



$$h_0 = \frac{2T\cos\theta}{\rho g r} = \frac{2\times 0.075\times 1}{10^3\times 10\times 10^{-4}} = 15\text{cm}$$

$$P_0V_0 = P\frac{100V_0}{101} \Rightarrow P = \frac{101}{100}P_0$$

$$P_0 - \frac{2T\cos\theta}{r} + \rho gh = P = \frac{101}{100}P_0$$

$$\Rightarrow -\rho gh_0 + \rho gh = \frac{P_0}{100}$$

$$\Rightarrow h = h_0 + \frac{P_0}{100\rho g}$$

$$= 15\text{ cm} + \frac{10^5}{100\times 10^3\times 10} = 25\text{cm}$$

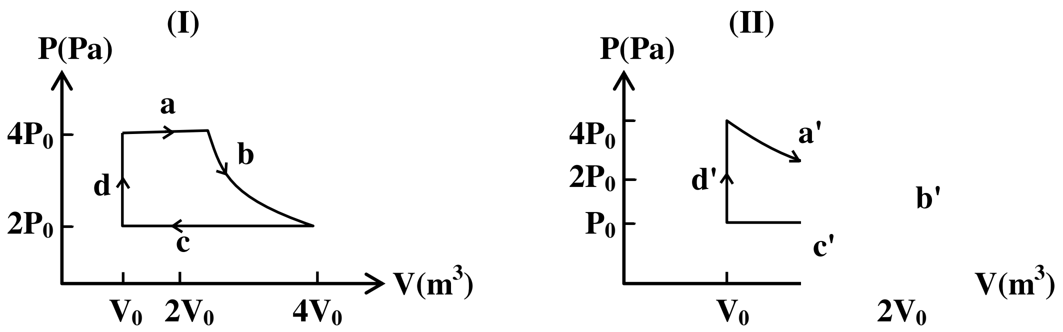
12. In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where $N(t)$ is the number of radioactive nuclei at time t. Two radioactive sources, S_1 and S_2 have same activity at time $t = 0$. At a later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. When S_1 and S_2 have just completed their 3rd and 7th half-lives, respectively, the ratio A_1/A_2 is ___________.

Ans. (16)

		S_1	S_2
Sol.	$t = 0$	A_0	A_0
	$t = \tau$	A_1	A_2

$$\frac{A_1}{A_2} = \frac{A_0(0.5)^{t/(t_{1/2})_1}}{A_0(0.5)^{t/(t_{1/2})_2}} = \frac{(0.5)^3}{(0.5)^7} = 2^4 = 16$$

13. One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the *P-V* diagrams below. In cycle I, processes *a, b, c* and *d* are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes *a′*, *b′*, *c′* and *d′* are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is W_I and that during cycle II is W_{II}. The ratio W_I / W_{II} is _______.



Ans. (2)

Sol. $\frac{W_I}{W_{II}} = \frac{4P_0V_0 + 8P_0V_0\ell n2 - 6P_0V_0 - 0}{4P_0V_0\ell n2 - 0 - P_0V_0 + 0}$

$= \frac{8\ell n2 - 2}{4\ell n2 - 1} = 2$

SECTION-4 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

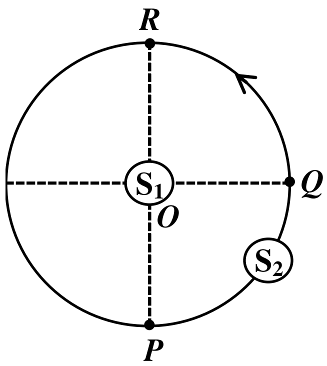
Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

PARAGRAPH-I

S_1 and S_1 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}\ ms^{-1}$ on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move along direction OP.

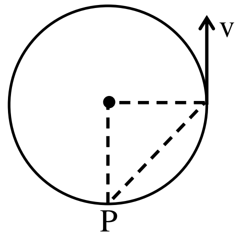
[Given: The speed of sound in air is 324 ms^{-1}]



14. When only S_2 is emitting sound and it is Q, the frequency of sound measured by the detector in Hz is ____________.

Ans. (648)

Sol.



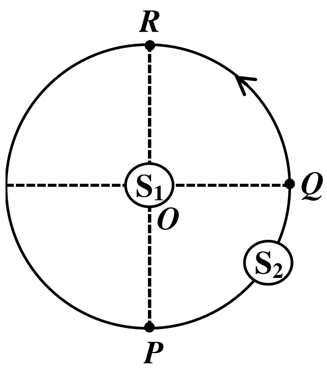
$$f' = \frac{C}{C + v\cos 45°} f$$

$$= \frac{324}{324 + 4\sqrt{2} \times \frac{1}{\sqrt{2}}} \times 656 = 648 \text{ Hz}$$

PARAGRAPH-I

S_1 and S_1 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}\ \text{ms}^{-1}$ on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move along direction OP.

[Given: The speed of sound in air is 324 ms^{-1}]



15. Consider both sources emitting sound. When S_2 is at R and S_1 approaches the detector with a speed 4 ms^{-1}, the beat frequency measured by the detector is _______Hz.

Ans. (8.2)

Sol. $f_{\text{P from } S_2} = 656\text{Hz}$

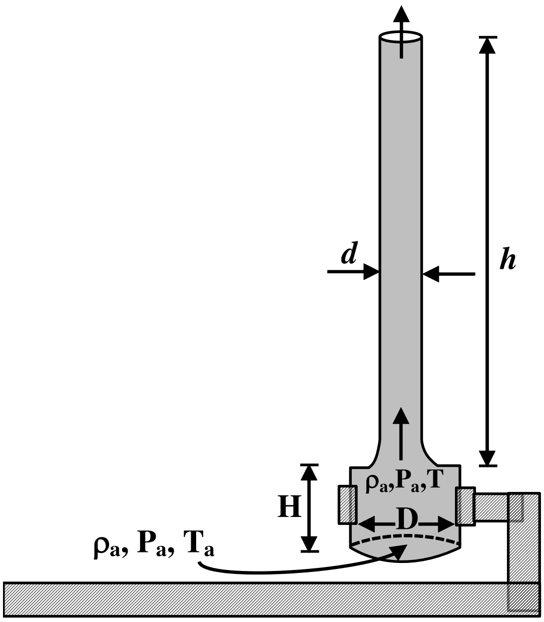
$$f_{\text{P from } S_1} = \frac{C}{C-V} f = \frac{656 \times 324}{324-4} = 664.2$$

$\Delta f = 664.2 – 656 = 8.2\text{Hz}$

PARAGRAPH-II

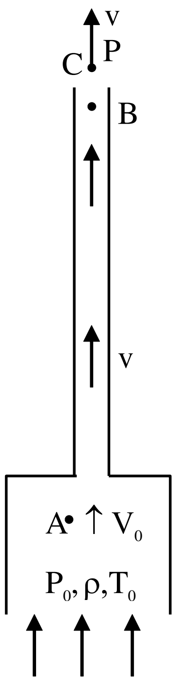
A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_a and its temperature becomes $T = 360$ K. The hot air with density ρ rises up a vertical chimney of diameter $d = 0.1$ m and height $h = 9$ m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density $\rho_a = 1.2$ kg m^{-3}, pressure P_a and temperature $T_a = 300$ K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects.

[Given: The acceleration due to gravity $g = 10$ ms^{-2} and $\pi = 3.14$]



16. Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is __________ gm s^{-1}.

Ans. (60.80, 60.81)



Sol.

$\rho_0 T_0 = \rho T$

$\Rightarrow 1.2 \times 300 = \rho(360) \therefore \rho = 1$

Between A & B

$$P_0 + \frac{1}{2}\rho V_0^2 = P + \frac{1}{2}\rho V^2 + \rho gh \quad ....(1)$$

$$\frac{\pi D^2}{4} V_0 = \frac{\pi d^2}{4} V \quad ....(2)$$

Between B & C

$$P + \frac{1}{2}\rho V^2 = P_0 - \rho_0 g(H+h) + \frac{1}{2}\rho V^2 \quad .....(3)$$

from (1) & (2) :

$$\Rightarrow P_0 + \frac{1}{2}\rho\left(V\frac{d^2}{D^2}\right)^2 = P + \frac{1}{2}\rho V^2 + \rho gh$$

$$\Rightarrow \rho_0 g(H+h) = \frac{1}{2}\rho V^2\left[1 - \frac{d^4}{D^4}\right] + \rho gh$$

$$\Rightarrow V^2 \simeq \frac{2\rho_0}{\rho} g(H+h) - 2gh$$

$= 2 \times 1.2 \times 10 \times 10 - 2 \times 10 \times 9$

$= 240 - 180 = 60 \quad \therefore V = \sqrt{60} m/s$

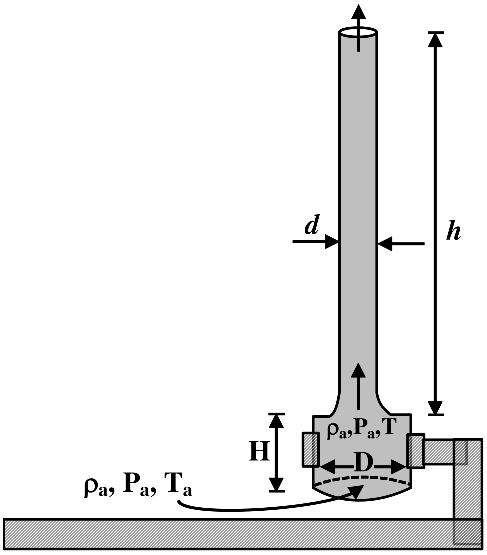
$$Q_m = \rho \frac{\pi d^2}{4} V = 1 \times \frac{\pi}{4} \times 10^{-2} \times \sqrt{60} \simeq 60.80$$

Ans. 60.80 to 60.81

PARAGRAPH-II

A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_a and its temperature becomes $T = 360$ K. The hot air with density ρ rises up a vertical chimney of diameter $d = 0.1$ m and height $h = 9$ m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density $\rho_a = 1.2$ kg m^{-3}, pressure P_a and temperature $T_a = 300$ K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects.

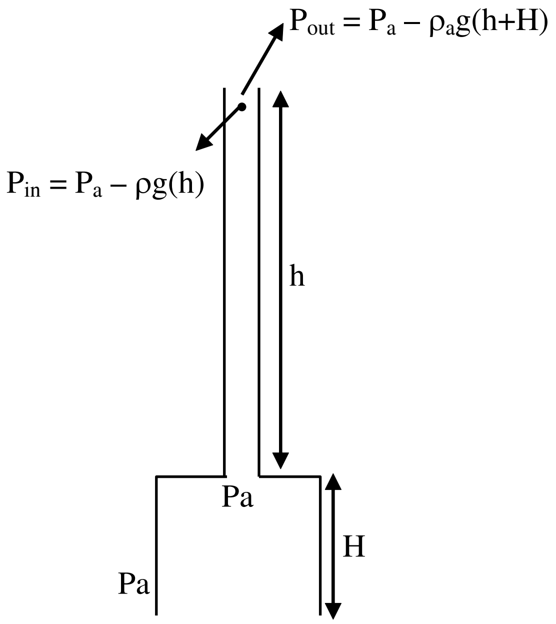
[Given: The acceleration due to gravity $g = 10$ ms^{-2} and $\pi = 3.14$]



17. When the chimney is closed using a cap at the top, a pressure difference ΔP develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of ΔP is ______ Nm^{-2}.

Ans. (30)

Sol.



P = constant

$\Rightarrow \rho_a T_a = \rho T$

$1.2 \times 300 = \rho \times 360$

$\rho = 1 \text{ kg/m}^3$

$\Delta P = \rho_a g\ (h + H) - \rho g h$

$= 1.2 \times 10 \times 10 - 1 \times 10 \times 9$

$= 120 - 90 = 30 \text{ N/m}^2$

CHEMISTRY

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

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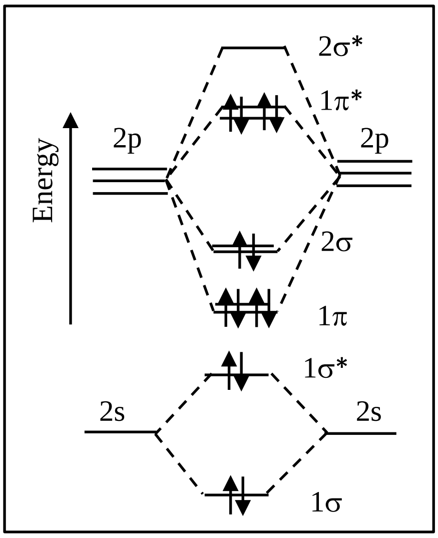
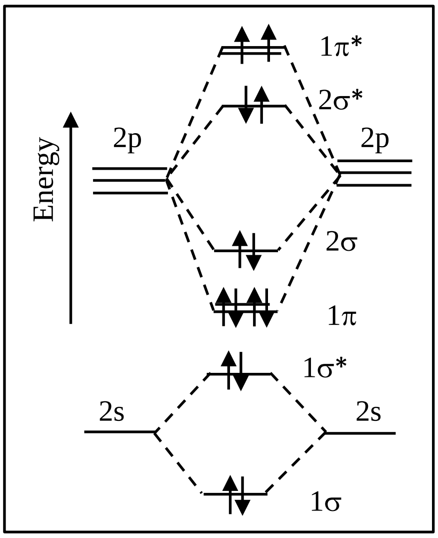
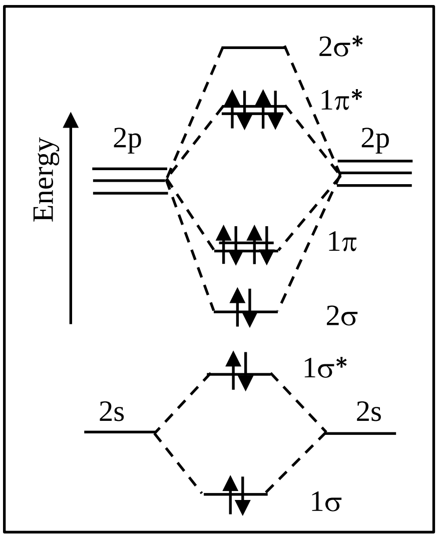
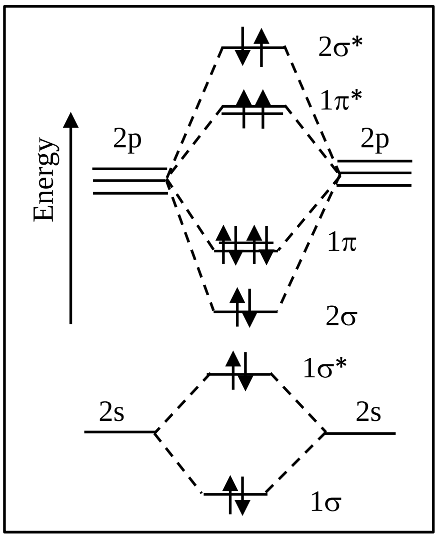
1. The correct molecular orbital diagram for F_2 molecule in the ground state is

(A)


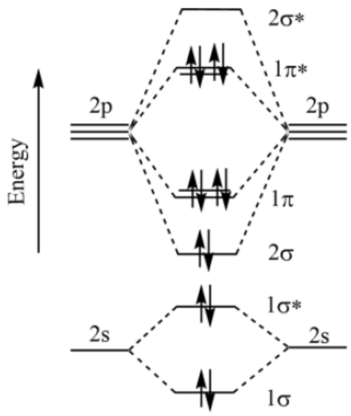
(B)


(C)


(D)


Ans. **(C)**

Sol. F_2 (18 e^-)



Naming of molecular orbitals are as per preference of formation of σ & π bonds respectively.

2. Consider the following statements related to colloids.

(I) Lyophobic colloids are **not** formed by simple mixing of dispersed phase and dispersion medium.

(II) For emulsions, both the dispersed phase and the dispersion medium are liquid.

(III) Micelles are produced by dissolving a surfactant in any solvent at any temperature.

(IV) Tyndall effect can be observed from a colloidal solution with dispersed phase having the same refractive index as that of the dispersion medium.

The option with the correct set of statements is

(A) (I) and (II) (B) (II) and (III) (C) (III) and (IV) (D) (II) and (IV)

Ans. **(A)**

Sol. (I) As in Lyophobic colloids there is no interaction between dispersed phase and dispersion medium, special methods are used for preparation, simple mixing will not form colloid.

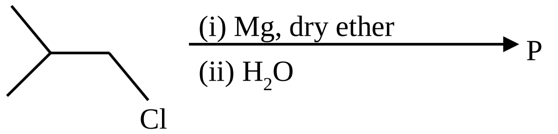
(II) Emulsions are liquid in liquid type colloids.

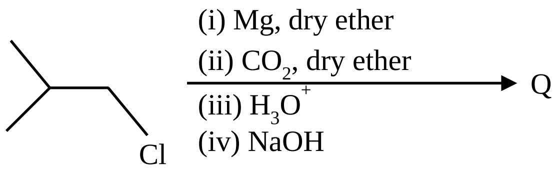
(III) Dissolving surfactant in a proper solvent will only form micelles at temperature above Kraft's temperature.

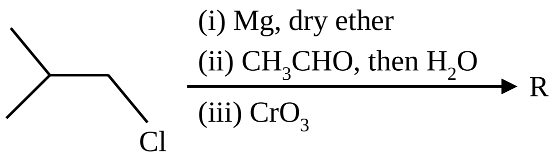
(IV) For Tyndall effect there must be a large difference in refractive index between dispersed phase and dispersion medium in order to have diffraction of light.

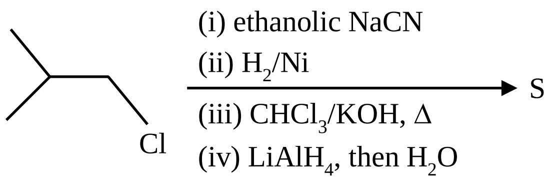
Hence ans (I) & (II) are correct.

3. In the following reactions, **P**, **Q**, **R**, and **S** are the major products.









The correct statement about **P**, **Q**, **R**, and **S** is

(A) **P** is a primary alcohol with four carbons.

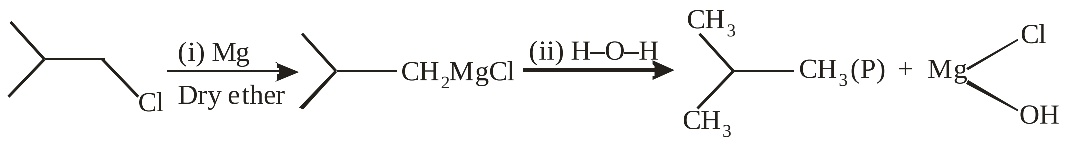
(B) **Q** undergoes Kolbe's electrolysis to give an eight-carbon product.

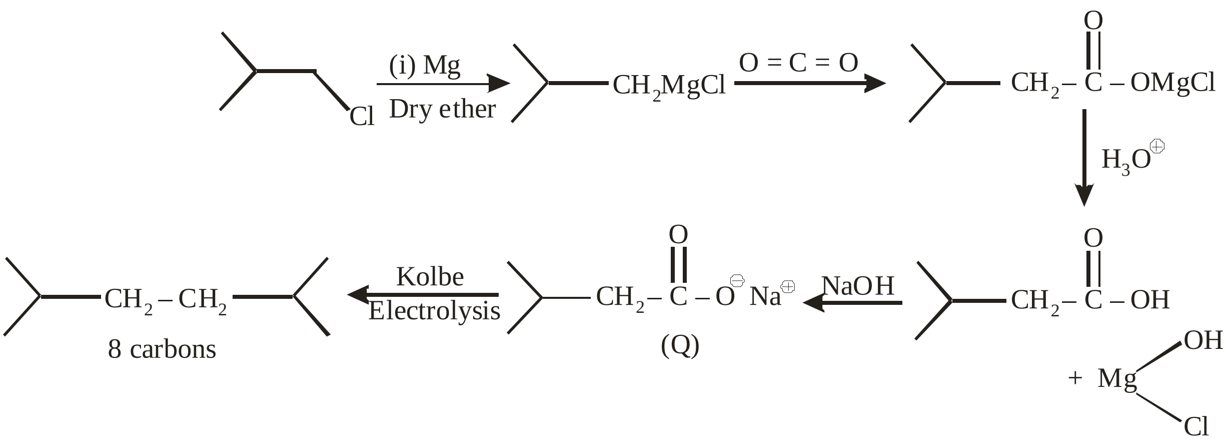
(C) **R** has six carbons and it undergoes Cannizzaro reaction.

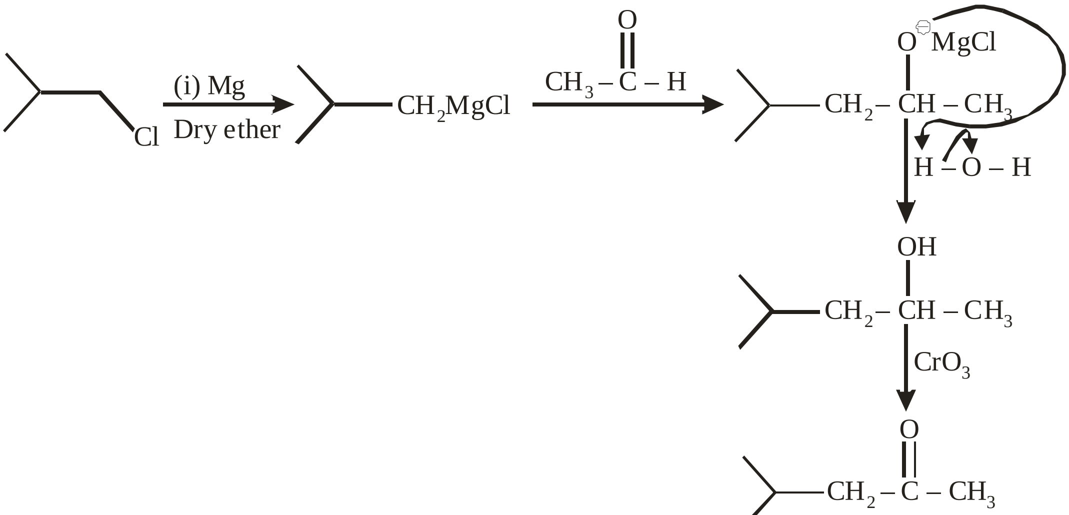
(D) **S** is a primary amine with six carbons.

Ans. **(B)**

Sol.

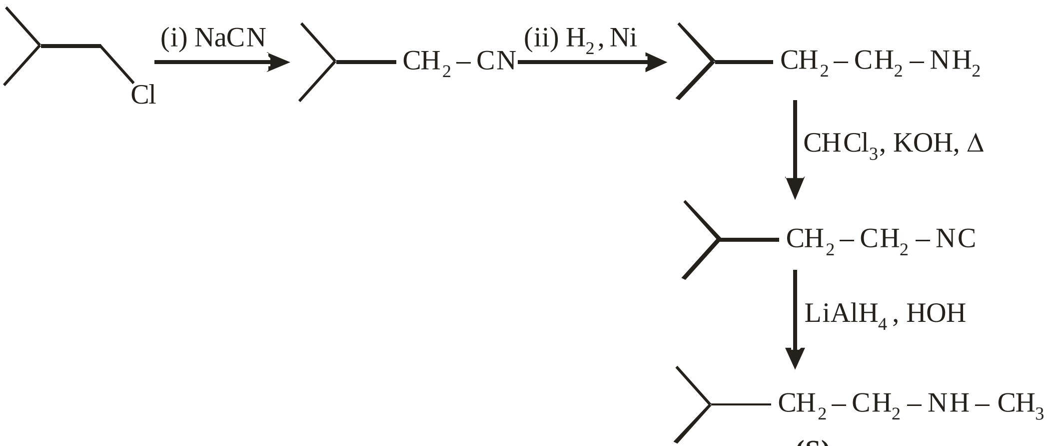






(R)

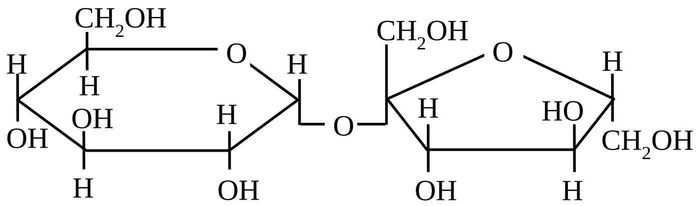
It does not give Cannizaro reaction

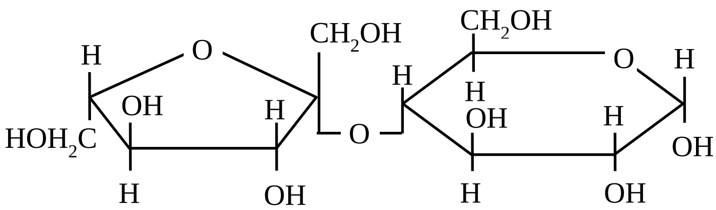


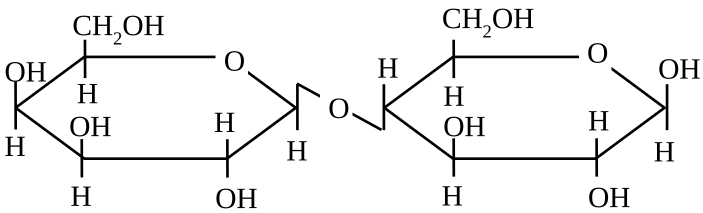
(S)

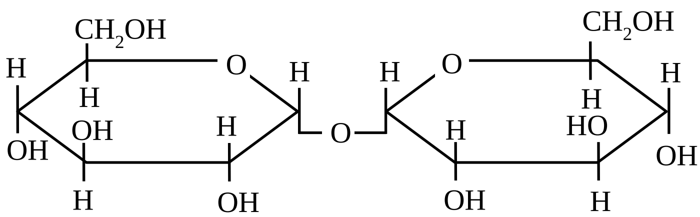
It's secondary amine

4. A disaccharide **X** cannot be oxidised by bromine water. The acid hydrolysis of **X** leads to a laevorotatory solution. The disaccharide **X** is

(A) 

(B) 

(C) 

(D) 

Ans. (A)

Sol. Sucrose $\xrightarrow{H_3O^+}$ Glucose + Fructose

Specific rotation + 52.5°   –92° (mixture of products is laevorotatory)

Sucrose $\xrightarrow{Br_2+H_2O}$ No reaction

BCD $\Rightarrow$ reducing sugars, will get oxidized by $Br_2 + H_2O$

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

choosing any other option(s) will get –2 marks.

---

5. The complex(es), which can exhibit the type of isomerism shown by $[Pt(NH_3)_2Br_2]$, is(are)

$[en = H_2NCH_2CH_2NH_2]$

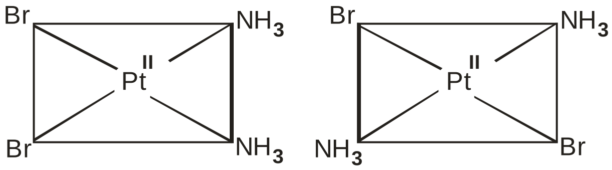
(A) $[Pt(en)(SCN)_2]$ (B) $[Zn(NH_3)_2Cl_2]$

(B) (C) $[Pt(NH_3)_2Cl_4]$ (D) $[Cr(en)_2(H_2O)(SO_4)]^+$

Ans. **(C,D)**

Sol. $[\overset{+2}{Pt}(NH_3)_2Br_2]$

Hybridisation : dsp^2 , geometry : square planar

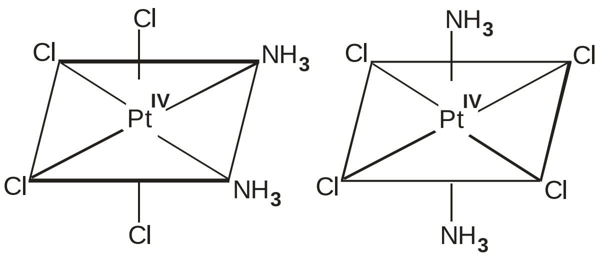


cis trans

(A) $[Pt(en)(SCN)_2]$: square planar, cis–trans not possible

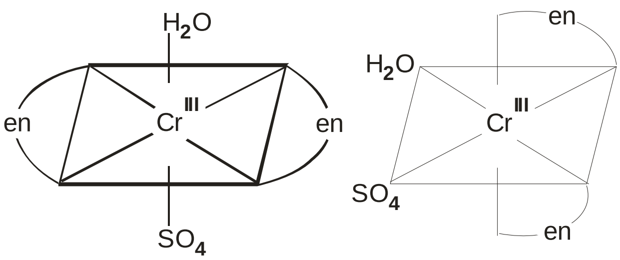
(B) $[Zn(NH_3)_2Cl_2]$: tetrahedral, cis–trans not possible

(C) $[Pt(NH_3)_2Cl_4]$: octahedral, cis–trans possible



cis trans

(D) $[Cr(en)_2(H_2O)SO_4]^+$: Octahedral



trans cis

6. Atoms of metals x, y, and z form face-centred cubic (fcc) unit cell of edge length L_x, body-centred cubic (bcc) unit cell of edge length L_y, and simple cubic unit cell of edge length L_z, respectively.

If $r_z = \frac{\sqrt{3}}{2} r_y ; r_y = \frac{8}{\sqrt{3}} r_x ; M_z = \frac{3}{2} M_y$ and $M_z = 3M_x$, then the correct statement (s) is (are)

[Given : M_x, M_y, and M_z are molar masses of metals x, y, and z, respectively.

r_x, r_y, and r_z are atomic radii of metals x, y, and z, respectively.]

(A) Packing efficiency of unit cell of x > Packing efficiency of unit cell of y > Packing efficiency of unit cell of z

(B) $L_y > L_z$

(C) $L_x > L_y$

(D) Density of x > Density of y

Ans. (A,B,D)

Sol.

Element	X	Y	Z
Packing	FCC	BCC	Primitive
Edge	L_x	L_y	L_z
Relation between edge length and radius	$L_x = 2\sqrt{2}r_x$	$L_y = \frac{4}{\sqrt{3}}r_y$	$L_z = 2r_z$
Packing fraction	$\frac{\pi}{3\sqrt{2}}$	$\frac{\sqrt{3}\pi}{8}$	$\frac{\pi}{6}$

Now, $r_y = \frac{8}{\sqrt{3}}r_x$ & $r_z = \frac{\sqrt{3}}{2}r_y = \frac{\sqrt{3}}{2}\times\frac{8}{\sqrt{3}}r_x \Rightarrow r_z = 4r_x$

So, $L_x = 2\sqrt{2}\,r_x$, $L_y = \frac{4}{\sqrt{3}}\times\frac{8}{\sqrt{3}}r_x$, $L_z = 8r_x$

$L_x = 2\sqrt{2}\,r_x$, $L_y = \frac{32}{3}r_x$, $L_z = 8r_x$

So $L_y > L_z > L_x$

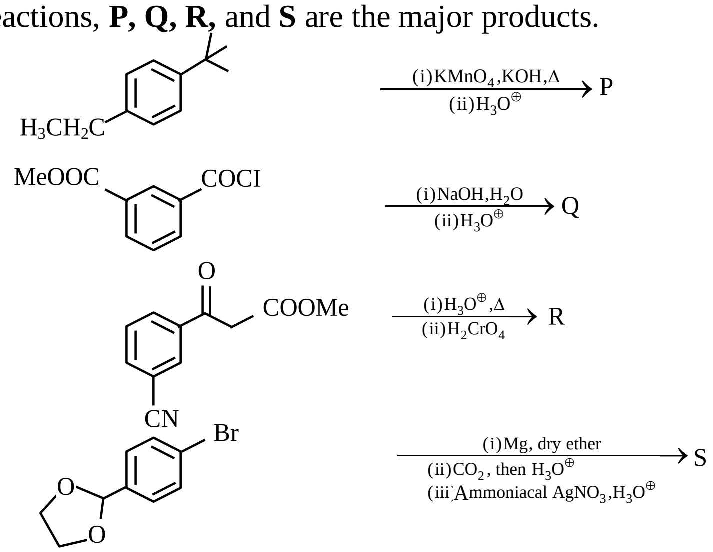
Density $\frac{4M_x}{L_x^3}, \frac{2\times M_y}{L_y^3}$

Now, $3M_x = \frac{3M_y}{2}$ or $M_x \times 2 = M_y$

$$\frac{\text{density}(x)}{\text{density}(y)} = \frac{4M_x}{2M_y}\times\frac{L_y^3}{L_x^3} = \frac{4M_x}{4M_x}\times\frac{\left(\frac{32}{3}\right)^3}{\left(2\sqrt{2}\right)^3}$$

Hence $d(x) > d(y)$

7. In the following reactions, **P, Q, R,** and **S** are the major products.



The correct statement (s) about **P, Q, R,** and **S** is (are)

(A) **P** and **Q** are monomers of polymers dacron and glyptal, respectively.

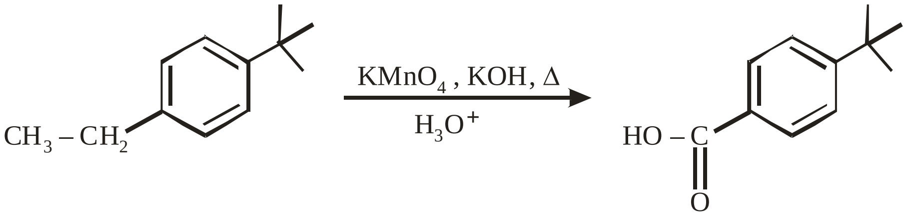
(B) **P, Q,** and **R** are dicarboxylic acids.

(C) Compounds **Q** and **R** are the same.

(D) **R** does **not** undergo aldol condensation and **S** does **not** undergo Cannizzaro reaction.

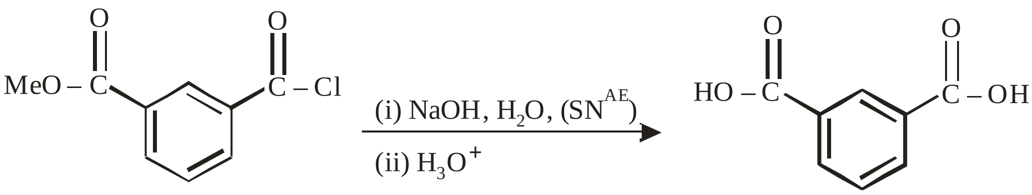
Ans. (C,D)

Sol.



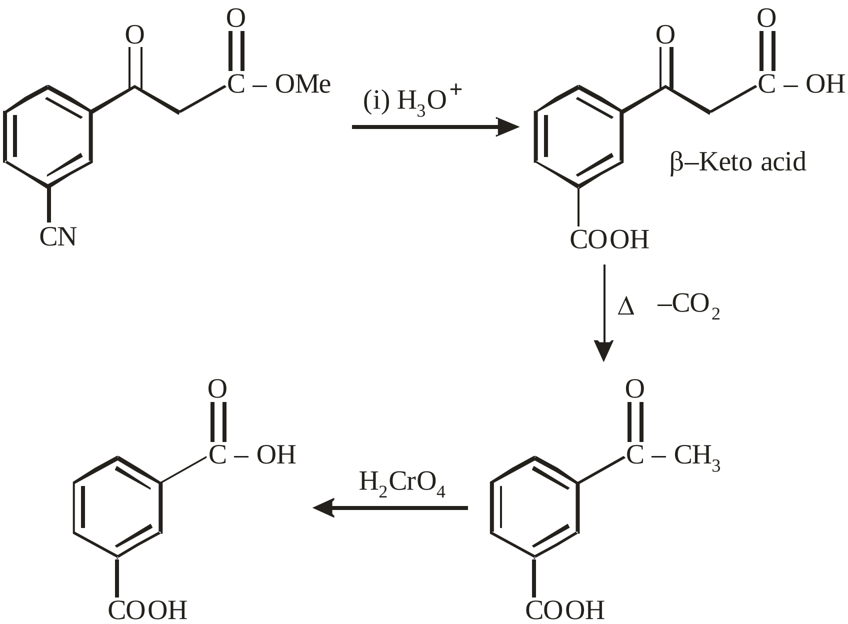
CH3 – CH2
KMnO4 , KOH, Δ
H3O+
HO – C
O

(P)



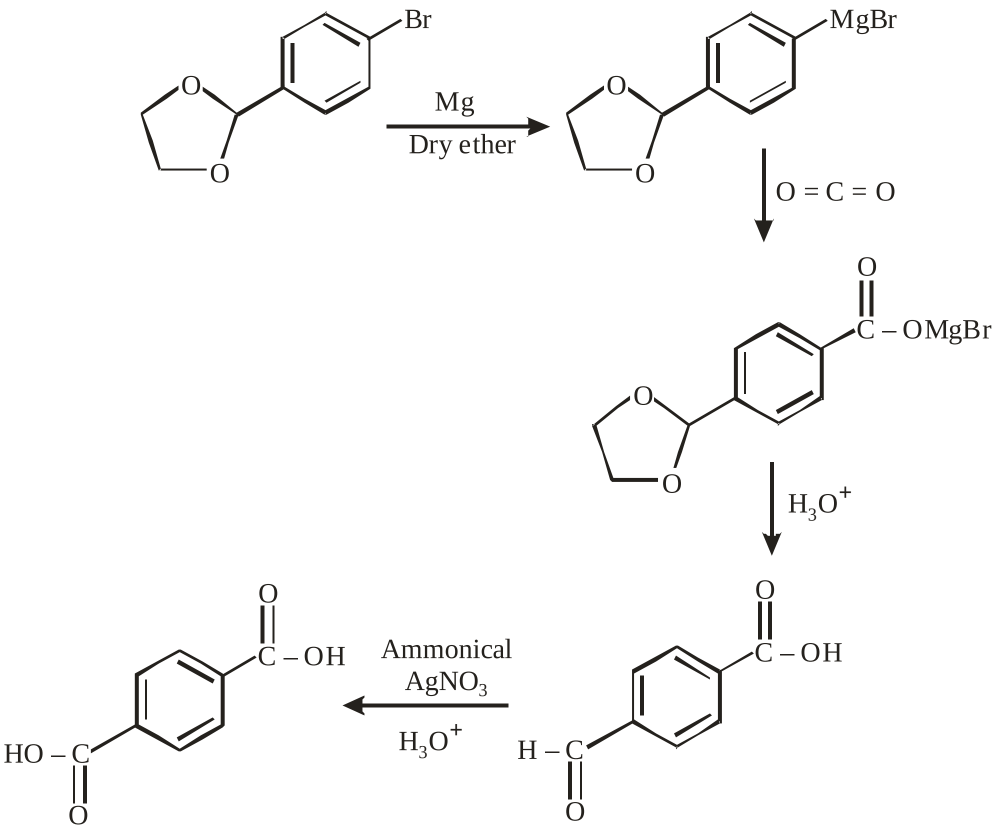
O
MeO – C
O
C – Cl
(i) NaOH, H2O, (SNAE)
(ii) H3O+
O
HO – C
O
C – OH

(Q)



O
O
C – OMe
CN
(i) H3O+
O
O
C – OH
β–Keto acid
COOH
Δ –CO2
O
C – CH3
COOH
H2CrO4
O
C – OH
COOH

(R)



SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases

---

8. H_2S (5 moles) reacts completely with acidified aqueous potassium permanganate solution. In this reaction, the number of moles of water produced is **x**, and the number of moles of electrons involved is **y**. The value of (**x** + **y**) is ____.

Ans. (18)

Sol. $2K\overset{+7}{Mn}O_4 + 5H_2S + 3H_2SO_4 \rightarrow K_2SO_4 + 2\overset{+2}{Mn}SO_4 + 5S + 8H_2O$

x = 8 (moles of H_2O produced)

y = 14 – 4 = 10 (number of electrons involved)

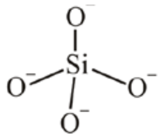
x + y = 10 + 8 = 18

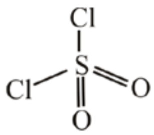
9. Among $[I_3]^+$, $[SiO_4]^{4-}$, SO_2Cl_2, XeF_2, SF_4, ClF_3, $Ni(CO)_4$, XeO_2F_2, $[PtCl_4]^{2-}$, XeF_4, and $SOCl_2$, the total number of species having sp^3 hybridised central atom is ______.

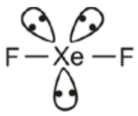
Ans. **(5)**

Sol.

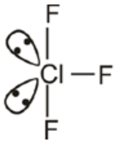
I_3^+ :  : sp^3

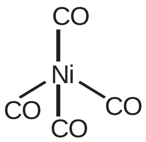
SiO_4^{4-} :  : sp^3

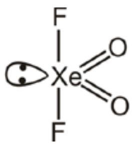
SO_2Cl_2 :  : sp^3

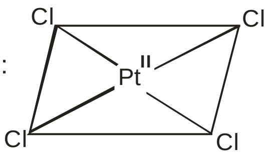
XeF_2 :  : sp^3d

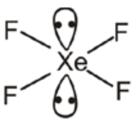
SF_4 :  : sp^3d

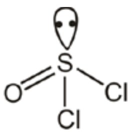
ClF_3 :  : sp^3d

$[Ni(CO)_4]$:  : sp^3

XeO_2F_2 :  : sp^3d

$[PtCl_4]^{2-}$:  : dsp^2

XeF_4 :  : sp^3d^2

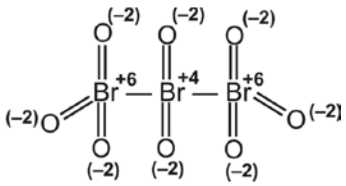
$SOCl_2$:  : sp^3

10. Consider the following molecules : Br_3O_8, F_2O, $H_2S_4O_6$, $H_2S_5O_6$, and C_3O_2.

Count the number of atoms existing in their zero oxidation state in each molecule. Their sum is____.

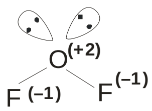
Ans. (6)

Sol. Br_3O_8



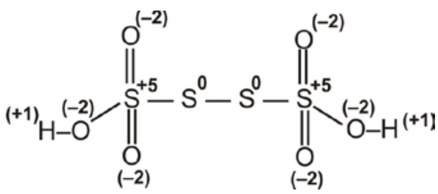
Number of atoms with zero oxidation state = 0

F_2O



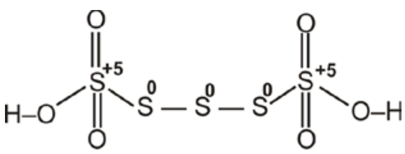
Number of atom with zero oxidation state = 0

$H_2S_4O_6$



Number of atoms with zero oxidation state = 2

$H_2S_5O_6$



Number of atoms where zero oxidation state = 3

C_3O_2

$O = C = \overset{(0)}{C} = C = O$

Number of atoms with zero oxidation state = 1

11. For He^+, a transition takes place from the orbit of radius 105.8 pm to the orbit of radius 26.45 pm. The wavelength (in nm) of the emitted photon during the transition is ____.

[**Use:**

Bohr radius, a = 52.9 pm

Rydberg constant, $R_H = 2.2 \times 10^{-18}$ J

Planck's constant, $h = 6.6 \times 10^{-34}$ J s

Speed of light, $c = 3 \times 10^{8}$ m s^{-1}]

Ans. (30)

Sol. For single electron system

$$r = 52.9 \times \frac{n^2}{Z} \text{ pm}$$

Given Z = 2 for He^+

$r_2 = 105.8$ pm

So $105.8 = 52.9 \times \frac{n_2^2}{2}$

$n_2 = 2$

$r_1 = 26.45$

So $26.45 = 52.9 \times \frac{n_1^2}{2}$

$n_1 = 1$

So transition is from 2 to 1.

Now $\frac{hc}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

So $\lambda = 30 \times 10^{-9}$ m = 30 nanometer.

Here 'R_H' is given in terms of energy value.

12. 50 mL of 0.2 molal urea solution (density = 1.012 g mL^{-1} at 300 K) is mixed with 250 mL of a solution containing 0.06 g of urea. Both the solutions were prepared in the same solvent. The osmotic pressure (in Torr) of the resulting solution at 300 K is ___.

[Use : Molar mass of urea = 60 g mol^{-1}; gas constant, R = 62 L Torr K^{-1} mol^{-1} ; Assume, $\Delta_{mix}H = 0$, $\Delta_{mix}V = 0$]

Ans. (682)

Sol. Weight of 50 ml 0.2 molal urea = V × d = 50 × 1.012 = 50.6 gm

Given 0.2 molal implies

1000 gm solvent has 0.2 moles urea

So weight of solution = 1000 + 0.2 × 60 = 1012 gm.

So wt. of urea in 50.6 gm solution = $\frac{12 \times 50.6}{1012}$ = 0.6 gm

Total urea = 0.6 + 0.06 = 0.66 gm

Total volume = 300 ml

Now, osmotic pressure $\pi = C \times R \times T = \frac{0.66 \times 62 \times 300}{60 \times 0.3}$ = 682 Torr.

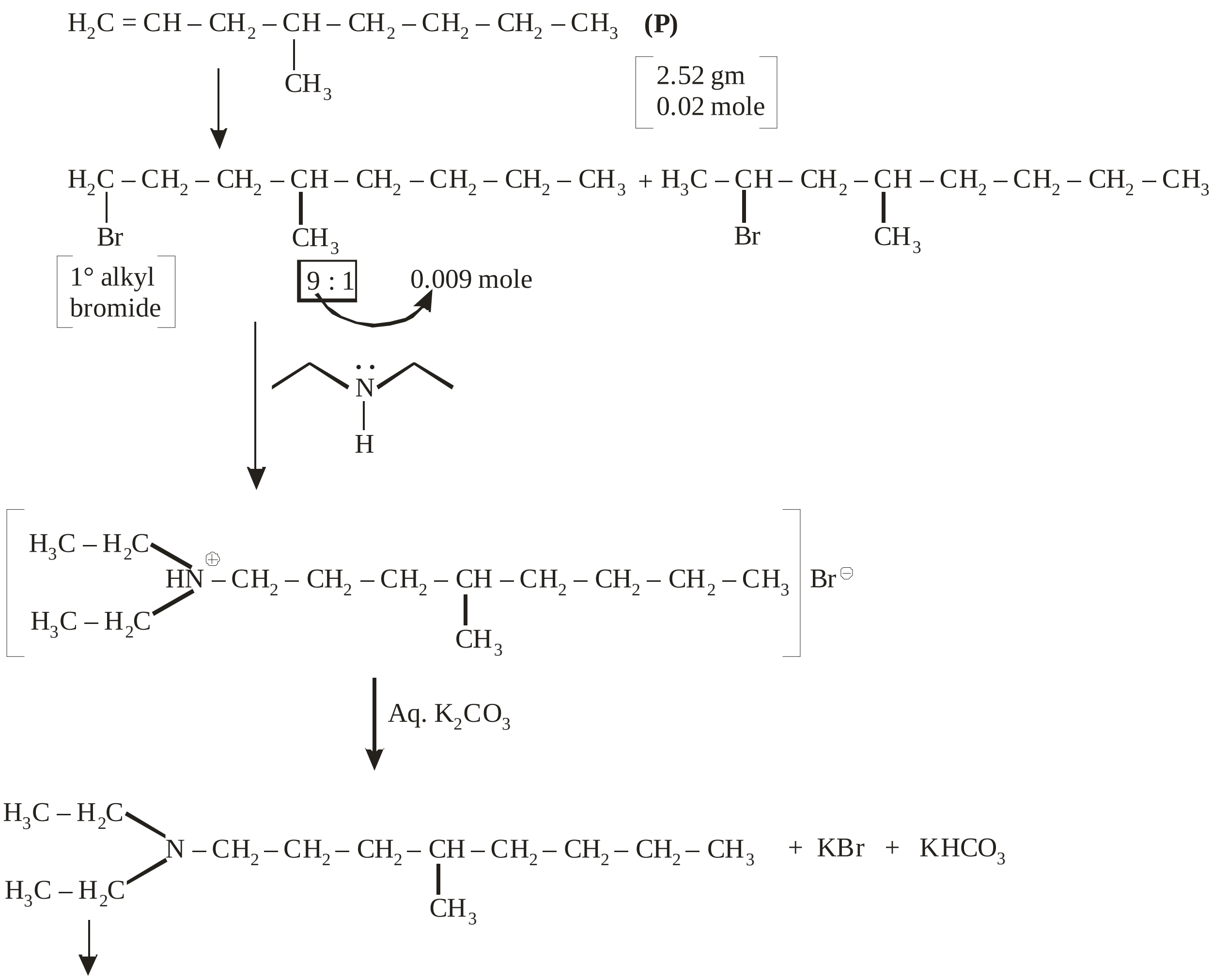
13. The reaction of 4-methyloct-ene (**P**, 2.52 g) with HBr in the presence of $(C_6H_5CO)_2O_2$ gives two isomeric bromides in a 9 : 1 ratio, with combined yield of 50%. Of these, the entire amount of the primary alkyl bromide was reacted with an appropriate amount of diethylamine followed by treatment with eq. K_2CO_3 to given a non-ionic product **S** in 100% yield.

The mass (in mg) of **S** obtained is __.

[Use molar mass (in g mol^{-1}) : H = 1, C = 12, N = 14, Br = 80]

Ans. (1791)

Sol.



mol. mass = 199 gm

(S) 0.009 mole

$= 199 \times 0.009 = 1791$ mg

SECTION-4 : (Maximum Marks : 12)

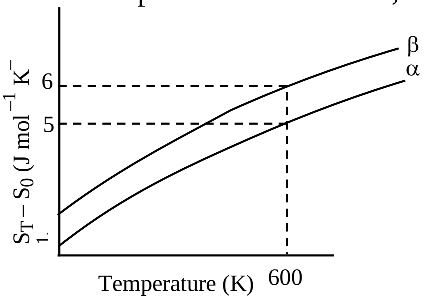
- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

"PARAGRAPH I"

The entropy versus temperature plot for phases α and β at 1 bar pressure is given.
S_T and S_0 are entropies of the phases at temperatures T and 0 K, respectively.



The transition temperature for α to β phase change is 600 K and $C_{P,\beta} - C_{P,\alpha} = 1$ J mol^{-1} K^{-1}. Assume $(C_{P,\beta} - C_{P,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{P,\alpha}$ and $C_{P,\beta}$ are heat capacities of α and β phases, respectively.

14. The value of entropy change, $S_\beta - S_\alpha$ (in J mol^{-1} K^{-1}), at 300 K is ___.
[Use : ln 2 = 0.69
Given : $S_\beta - S_\alpha = 0$ at 0 K]

Ans. **(0.31)**

Sol. At 1 bar

$$\alpha \longrightarrow \beta$$

$$S^o_{\alpha(600)} = S^o_{\alpha(300)} + C_{P(\alpha)}\ell n\frac{600}{300}$$

$$S^o_{\beta(600)} = S^o_{\beta(300)} + C_{P(\beta)}\ell n\frac{600}{300}$$

$$S^o_{\beta(600)} - S^o_{\alpha(600)} = S^o_{\beta(300)} - S^o_{\alpha(300)} + (C_{P(\beta)} - C_{P(\alpha)})\ \ell n 2$$

$$6 - 5 = S^o_{\beta(300)} - S^o_{\alpha(300)} + 1\times \ell n 2$$

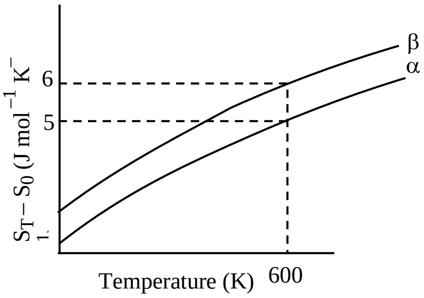
$$1 = S^o_{\beta(300)} - S^o_{\alpha(300)} + 0.69$$

$$\text{So } S^o_{\beta(300)} - S^o_{\alpha(300)} = 0.31$$

"PARAGRAPH I"

The entropy versus temperature plot for phases α and β 1 bar pressure is given.

S_T and S_0 are entropies of the phases at temperatures T and 0 K, respectively



The transition temperature for α to β phase change is 600 K and $C_{P,\beta} - C_{P,\alpha} = 1 \text{J mol}^{-1} \text{K}^{-1}$. Assume $(C_{P,\beta} - C_{P,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{P,\alpha}$ and $C_{P,\beta}$ are heat capacities of α and β phases, respectively.

15. The value of enthalpy change, $H_\beta - H_\alpha$ (in J mol^{-1}), at 300 K is __.

Ans. (300)

Sol. As the phase transition temperature is 600 K

So at 600 K $\Delta G°_{rxn} = 0$

So $\Delta H°_{reaction\ (600)} = T\ \Delta S°_{reaction\ (600)}$

$\Delta H°_{(600)} = 600 \times 1 = 600$ Joule/mole

So $\Delta H_{600} - \Delta H_{300} = \Delta C_P (T_2 - T_1)$

$\Delta H_{600} - \Delta H_{300} = 1 \times 300$

$\Delta H_{300} = \Delta H_{600} - 300 = 600 - 300 = 300$ Joule/mole.

"PARAGRAPH II"

A trinitro compound, 1, 3,5 tris-(4-nitrophenyl) benzene, on complete reaction with an excess of Sn/HCl gives major product, which on treatment with an excess of $NaNO_2$/HCl at 0°C provides **P** as the product. **P**, upon treatment with excess of H_2O at room temperature, gives the product **Q.** Bromination of **Q** in aqueous medium furnishes the product **R.** The compound **P** upon treatment with an excess of phenol under basic conditions gives the product **S.**

The molar mass difference between compounds **Q** and **R** is 474 mol^{-1} and between compounds **P** and **S** is 172.5 g mol^{-1}.

16. The number of heteroatoms present in one molecule of **R** is _____.

[Use: Molar mass (in g mol^{-1}): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5

Atoms other than C and H are considered as heteroatoms]

Ans. (9)

"PARAGRAPH II"

A trinitro compound, 1, 3,5 tris-(4-nitrophenyl) benzene, on complete reaction with an excess of Sn/HCl gives major product, which on treatment with an excess of $NaNO_2$/HCl at 0°C provides **P** as the product. **P,** upon treatment with excess of H_2O at room temperature, gives the product **Q.** Bromination of **Q** in aqueous medium furnishes the product **R.** The compound **P** upon treatment with an excess of phenol under basic conditions gives the product **S.**

The molar mass difference between compounds **Q** and **R** is 474 mol^{-1} and between compounds **P** and **S** is 172.5 g mol^{-1}.

17. The total number of carbon atoms and heteroatoms present in one molecule of **S** is _____.

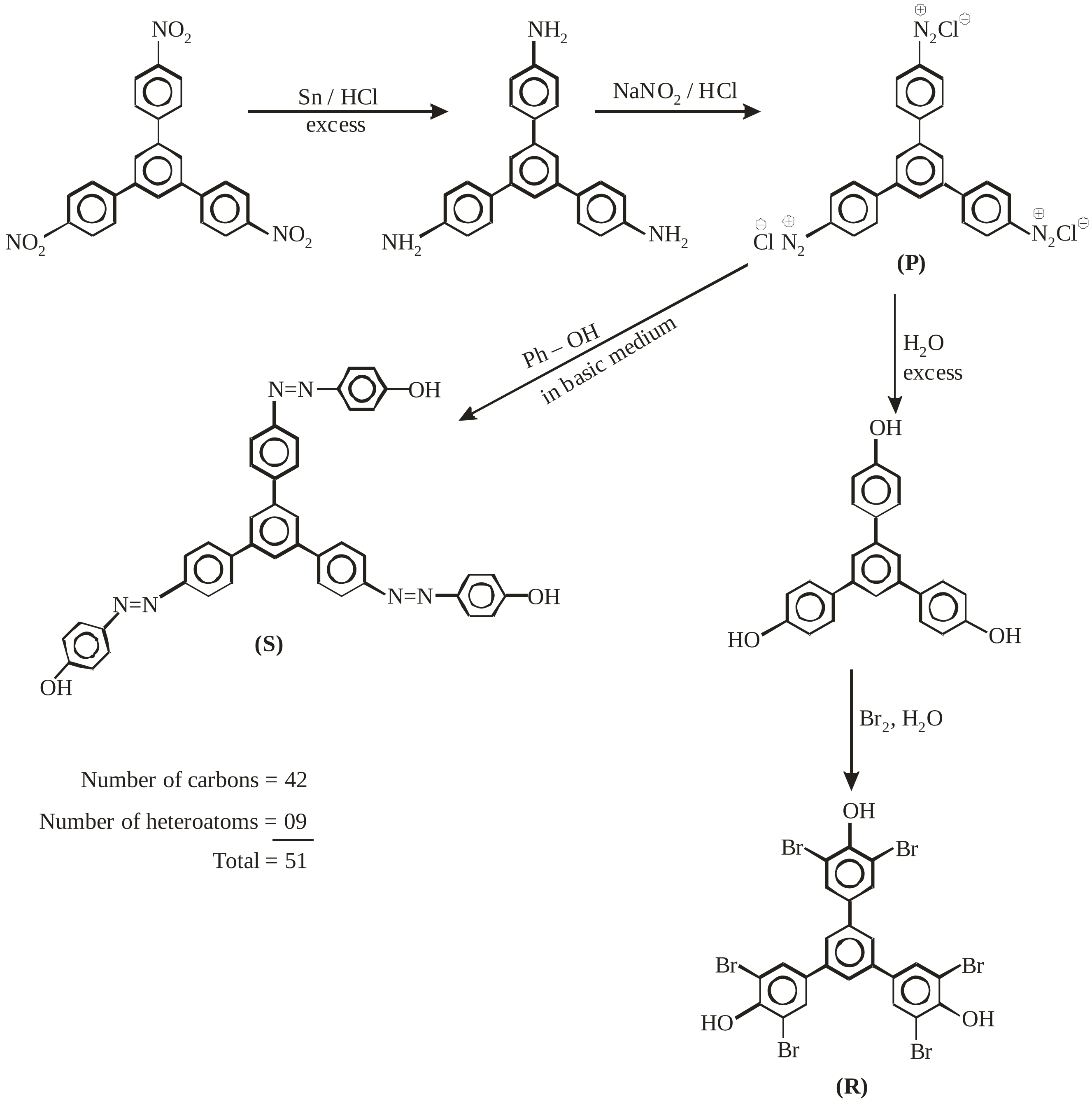
[Use: Molar mass in g mol^{-1}]: H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5

Atoms other than C and H are considered as heteroatoms

Ans. (51)

Sol.

Common solution for Q.no. 16 and 17



Number of carbons = 42

Number of heteroatoms = 09

Total = 51

Number of hetero atoms in R is 9

MATHEMATICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

---

1. Let $f:[(1,\infty)\to\mathbb{R}$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)dt = xf(x)-\frac{x^3}{3}, x\in[1,\infty)$.

Let e denote the base of the natural logarithm. Then the value of f(e) is

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4+e}{3}$

(C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

Ans. (C)

Sol. Diff. wr.t 'x'

$3f(x) = f(x) + xf'(x) - x^2$

$\frac{dy}{dx}-\left(\frac{2}{x}\right)y = x$

$IF = e^{-2\ell nx} = \frac{1}{x^2}$

$y\left(\frac{1}{x^2}\right) = \int x.\frac{1}{x^2}dx$

$y = x^2\ell nx + cx^2$

$\therefore\ y(1) = \frac{1}{3} \Rightarrow c = \frac{1}{3}$

$y(e) = \frac{4e^2}{3}$

2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is.

(A) $\frac{1}{3}$ (B) $\frac{5}{21}$ (C) $\frac{4}{21}$ (D) $\frac{2}{7}$

Ans. (B)

Sol. $P(H)=\frac{1}{3};P(T)=\frac{2}{3}$

Req. prob = P(HH or HTHH or HTHTHH or …….)

+ P(THH or THTHH or THTHTHH or ….)

$$=\frac{\frac{1}{3}\cdot\frac{1}{3}}{1-\frac{2}{3}\cdot\frac{1}{3}}+\frac{\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}{1-\frac{2}{3}\cdot\frac{1}{3}}=\frac{5}{21}$$

3. For any $y\in\mathbb{R}$, let $\cot^{-1}(y)\in(0,\pi)$ and $\tan^{-1}(y)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\cot^{-1}\left(\frac{9-y^2}{6y}\right)=\frac{2\pi}{3}$ for $0<|y|<3$, is equal to

(A) $2\sqrt{3}-3$ (B) $3-2\sqrt{3}$

(C) $4\sqrt{3}-6$ (D) $6-4\sqrt{3}$

Ans. (C)

Sol. Case-I : $y\in(-3,0)$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\pi+\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}$$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=-\frac{\pi}{3}$$

$$y^2-6\sqrt{3}y-9=0\Rightarrow y=3\sqrt{3}-6\ (\because\ y\in(-3,0))$$

Case-I : $y\in(0,3)$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}\Rightarrow\sqrt{3}y^2+6y-9\sqrt{3}=0$$

$y=\sqrt{3}$ or $y=-3\sqrt{3}$ (rejected)

sum $=\sqrt{3}+3\sqrt{3}-6=4\sqrt{3}-6$

4. Let the position vectors of the points P,Q,R and S be $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=3\hat{i}+6\hat{j}+3\hat{k}$, $\vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively. Then which of the following statements is true?

(A) The points P,Q,R and S are **NOT** coplanar

(B) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b}\times\vec{d}$ is 95

Ans. (B)

Sol. $P\left(\hat{i}+2\hat{j}-5\hat{k}\right)=P(\vec{a})$

$Q\left(3\hat{i}+6\hat{j}+3\hat{k}\right)=Q\left(\vec{b}\right)$

$R\left(\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}\right)=R(\vec{c})$

$S\left(2\hat{i}+\hat{j}+\hat{k}\right)=S\left(\vec{d}\right)$

$\frac{\vec{b}+2\vec{d}}{3}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}$

$\frac{5\vec{c}+4\vec{a}}{9}=\frac{21\hat{i}+24\hat{j}+15\hat{k}}{9}$

$\Rightarrow \frac{\vec{b}+2\vec{d}}{3}=\frac{5\vec{c}+4\vec{a}}{9}$

so [B] is correct.

option –D

$\left|\vec{b}\times\vec{d}\right|^2=\left|\vec{b}\right|\left|\vec{d}\right|^2-\left(\vec{b}.\vec{d}\right)^2$

$= (9 + 36 + 9)\,(4 + 1 + 1) - (6 + 6 + 3)^2$

$= 54 \times 6 - (15)^2$

$= 324 - 225$

$= 99$

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks : –2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get –2 marks.

---

5. Let $M = (a_{ij}),\ i, j \in \{1, 2, 3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j+1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is (are) true ?

(A) M is invertible

(B) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\} \neq \{\mathbf{0}\}$, where $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the 3×3 identity matrix

Ans. (B,C)

Sol. $M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$|M| = -1 + 1 = 0 \Rightarrow$ M is singular so non-invertible

(B) $M\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$

$\left.\begin{array}{l} a_1 + a_2 + a_3 = -a_1 \\ a_1 + a_3 = -a_2 \\ a_2 = -a_3 \end{array}\right\} \Rightarrow a_1 = 0$ and $a_2 + a_3 = 0$ infinite solutions exists [B] is correct.

Option (D)

$M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

$|M - 2I| = 0 \Rightarrow$ [D] is wrong

Option (C) :

$MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x + y + z = 0$

$x + z = 0$

$y = 0$

$\therefore$ Infinite solution

[C] is correct

6. Let $f : (0,1) \to \mathbb{R}$ be the function defined as $f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right)$, where $[x]$ denotes the greatest integer less than or equal to x. Then which of the following statements is(are) true?

(A) The function f is discontinuous exactly at one point in (0,1)

(B) There is exactly one point in (0,1) at which the function f is continuous but **NOT** differentiable

(C) The function f is **NOT** differentiable at more than three points in (0,1)

(D) The minimum value of the function f is $-\frac{1}{512}$

Ans. (A,B)

Sol. $f(x)=\begin{cases} 0 & ; \quad 0<x<\frac{1}{4} \\ \left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & ; \quad \frac{1}{4}\le x<\frac{1}{2} \\ 2\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & ; \quad \frac{1}{2}\le x<\frac{3}{4} \\ 3\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & ; \quad \frac{3}{4}\le x<1 \end{cases}$

f(x) is discontinuous at $x=\frac{3}{4}$ only

$$f'(x)=\begin{cases} 0 & ; \quad 0<x<\frac{1}{4} \\ 2\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)+\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{1}{4}<x<\frac{1}{2} \\ 4\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)+2\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{1}{2}<x<\frac{3}{4} \\ 6\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)+3\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{3}{4}<x<1 \end{cases}$$

f(x) is non-differentiable at $x=\frac{1}{2}$ and $\frac{3}{4}$

minimum values of f(x) occur at $x=\frac{5}{12}$ whose value is $-\frac{1}{432}$

7. Let S be the set of all twice differentiable functions f from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2f}{dx^2}(x)>0$ for all $x \in (-1,1)$. For $f \in S$, let X_f be the number of points $x \in (-1,1)$ for which f (x) = x. Then which of the following statements is(are) true?

(A) There exists a function $f \in S$ such that $X_f = 0$

(B) For every function $f \in S$, we have $X_f \le 2$

(C) There exists a function $f \in S$ such that $X_f = 2$

(D) There does **NOT** exist any function f in S such that $X_f = 1$

Ans. (A,B,C)

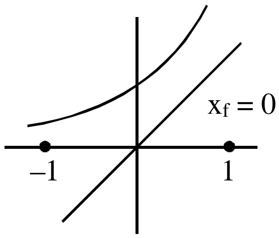
Sol. S = Set of all twice differentiable functions $f : R \rightarrow R$

$\frac{d^2f}{dx^2} > 0$ in (–1, 1)

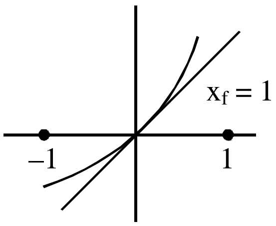
Graph 'f' is Concave upward.

Number of solutions of $f(x) = x \rightarrow x_f$

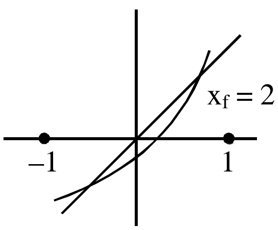
(1)



(2)



(3)



$\Rightarrow$ Graph of y = f(x) can intersect graph of y = x at atmost two points $\Rightarrow 0 \le x_f \le 2$

<u>Aliter</u>

$\frac{d^2f(x)}{dx^2} > 0$

Let $\phi(x) = f(x) - x$

$\phi''(x) > 0$

$\therefore$ $\phi'(x) = 0$ has atmost 1 root in $x \in (-1, 1)$

$\therefore$ $\phi(x) = 0$ has atmost 2 roots in $x \in (-1, 1)$

$\therefore$ $x_f \le 2$

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases

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8. For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}}dt \text{ is}$$

Ans. (0)

Sol. $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{t-\cos t}}{1+t^{2023}}dt$

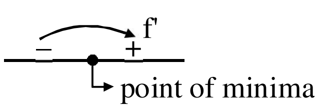
$$f'(x) = \frac{e^{x\tan^{-1}x-\cos(x\tan^{-1}x)}}{1+(x\tan^{-1}x)^{2023}} \cdot \left(\frac{x}{1+x^2} + \tan^{-1}x\right)$$

For $x < 0,\ \tan^{-1}x \in \left(-\frac{\pi}{2}, 0\right)$

For $x \geq 0,\ \tan^{-1}x \in \left[0, \frac{\pi}{2}\right)$

$\Rightarrow x\tan^{-1}x \geq 0\ \forall\ x \in R$

$$\text{And } \frac{x}{1+x^2} + \tan^{-1}x = \begin{cases} >0 & \text{For } x > 0 \\ <0 & \text{For } x < 0 \\ 0 & \text{For } x = 0 \end{cases}$$



Hence minimum value is $f(0) = \int_0^0 = 0$

9. For $x \in \mathbb{R}$, let y(x) be a solution of the differential equation

$$(x^2 - 5)\frac{dy}{dx} - 2xy = -2x(x^2 - 5)^2 \text{ such that } y(2) = 7.$$

Then the maximum value of the function y(x) is

Ans. (16)

Sol. $\frac{dy}{dx} - \frac{2x}{x^2 - 5}y = -2x(x^2 - 5)$

$$IF = e^{-\int \frac{2x}{x^2-5}dx} = \frac{1}{(x^2 - 5)}$$

$$y.\frac{1}{x^2 - 5} = \int -2x.dx + c$$

$$\Rightarrow \frac{y}{x^2 - 5} = -x^2 + c$$

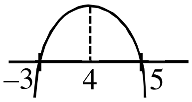
$x = 2, y = 7$

$$\frac{7}{-1} = -4 + c \Rightarrow c = -3$$

$y = -(x^2 - 5)(x^2 + 3)$

put $x^2 = t > 0$

$y = -(t - 5)(t + 3)$



$y_{max} = 16$ when $x^2 = 1$

$y_{max} = 16$

10. Let X be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to

Ans. (31)

Sol. No. of elements in X which are multiple of 5

$$\left.\begin{array}{l}
\underbrace{_\,_\,_\,_}_{1,2,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 3} = 4 \\
\underbrace{_\,_\,_\,_}_{1,4,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 2} = 12 \\
\underbrace{_\,_\,_\,_}_{4,2,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 3} = 4 \\
\underbrace{_\,_\,_\,_}_{2,2,4,4}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 6 \\
\underbrace{_\,_\,_\,_}_{1,2,4,4}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 2} = 12
\end{array}\right\} \text{Total} = 38$$

Among these 38 elements, let us calculate when element is not divisible by 20

$$\left.\begin{array}{l}
\underbrace{_\,_\,_}_{2,2,2}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \frac{\lfloor 3}{\lfloor 3} = 1 \\
\underbrace{_\,_\,_}_{2,2,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \frac{\lfloor 3}{\lfloor 2} = 3 \\
\underbrace{_\,_\,_}_{2,4,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \frac{\lfloor 3}{\lfloor 2} = 3
\end{array}\right\} \text{Total} = 7$$

$$\therefore\ p = \frac{38-7}{38} \quad \therefore\ 38p = 31$$

11. Let $A_1, A_2, A_3, \ldots\ldots, A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for i = 1,2,....,8. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2. \cdot\cdot\cdot\cdot\cdot\ PA_8$, is

Ans. (512)

Sol. $z^8 - 2^8 = (z-2)(z-\alpha)(z-\alpha^2)\ldots(z-\alpha^7)$

Put $z = 2e^{i\theta}$

$2^8(e^{i8\theta} - 1) = (2e^{i\theta} - 2)(2e^{i\theta} - \alpha)\ldots..(2e^{i\theta} - \alpha^7)$

Take mod

$2^8|e^{i8\theta} - 1| = PA_1\ PA_2 \ldots PA_8$

$2^8|2\sin 4\theta| = PA_1\ PA_2 \ldots PA_8$

$(PA_1 \cdot PA_2 \ldots PA_8)_{max} = 512$

12. Let $R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0, 3, 5, 7, 11, 13, 17, 19\} \right\}$. Then the number of invertible matrices in R is

Ans. (3780)

Sol. Let us calculate when $|R| = 0$

Case-I $ad = bc = 0$

Now $ad = 0$

$\Rightarrow$ Total – (When none of a & d is 0)

$= 8^2 - 1 = 15$ ways

Similarly $bc = 0 \Rightarrow 15$ ways

$\therefore 15 \times 15 = 225$ ways of $ad = bc = 0$

Case-II $ad = bc \neq 0$

either $a = d = b = c$ OR $a \neq d,\ b \neq d$ but $ad = bc$

${}^7C_1 = 7$ ways ${}^7C_2 \times 2 \times 2 = 84$ ways

Total 91 ways

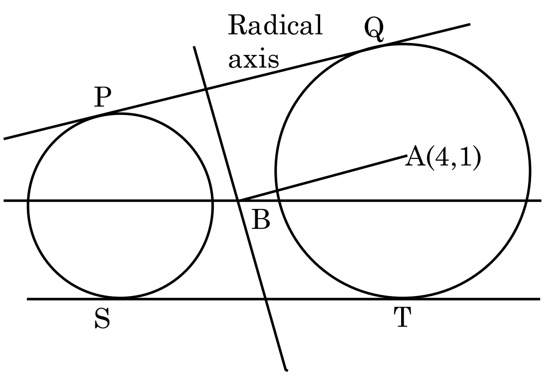
$\therefore |R| = 0$ in $225 + 91 = 316$ ways

$|R| \neq 0$ in $8^4 - 316 = 3780$

13. Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with center at the point $A = (4,1)$, where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If $AB = \sqrt{5}$, then the value of r^2 is

Ans. (2)

Sol.



Let C_2 $(x-4)^2 + (y-1)^2 = r^2$

radical axis $8x + 2y - 17 = 1 - r^2$

$8x + 2y = 18 - r^2$

$B\left(\frac{18-r^2}{8}, 0\right)$ A(4,1)

$AB = \sqrt{5}$

$$\sqrt{\left(\frac{18-r^2}{8}-4\right)^2+1} = \sqrt{5}$$

$r^2 = 2$

$\Rightarrow n = \sin\alpha + \cos\alpha$

SECTION-4 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

PARAGRAPH "I"

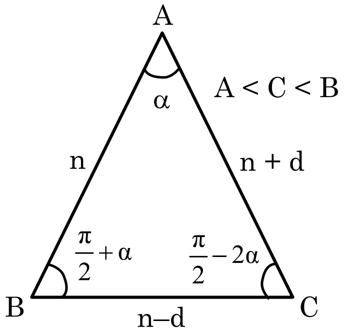
Consider on obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

14. Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

Ans. (1008.00)

Sol.



$n - d = 2\sin\alpha$ ...(1)

$n + d = 2\sin\left(\frac{\pi}{2}+\alpha\right)$

$\Rightarrow n + d = 2\cos\alpha$ ....(2)

$n = 2\sin\left(\frac{\pi}{2}-2\alpha\right)$

$\Rightarrow n = 2\cos 2\alpha$ ...(3)

$\Rightarrow 2\cos 2\alpha = \sin\alpha + \cos\alpha$

$\Rightarrow 2(\cos\alpha - \sin\alpha) = 1$

$\Rightarrow \sin 2\alpha = \frac{3}{4}$

Then, $a = \frac{1}{2}.n.(n+d).\sin\alpha = \frac{1}{2}.2\cos 2\alpha.2\cos\alpha.\sin\alpha$

$= \sin 2\alpha.\cos 2\alpha$

$= \frac{3}{4}\times\frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{16}$

$(64a)^2 = \left(64\times\frac{3\sqrt{7}}{16}\right)^2 = 16\times 9\times 7 = 1008$

PARAGRAPH "I"

Consider on obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

15. Then the inradius of the triangle ABC is

Ans. (0.25)

Sol. From above equation in Ques. 14

$$r = \frac{\Delta}{s} = \frac{1}{2}\frac{n(n+d)\sin\alpha}{\left(\frac{3n}{2}\right)}$$

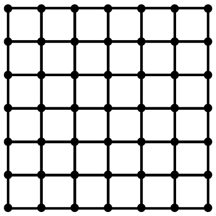
$$= \frac{(n+d).\sin\alpha}{3}$$

$$= \frac{2\cos\alpha.\sin\alpha}{3} \quad \text{(from (2))}$$

$$r = \frac{\sin 2\alpha}{3} = \frac{1}{4}$$

PARAGRAPH "II"

Consider the 6 × 6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.

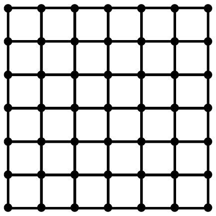


(There are two questions based on PARAGRAPH "II", the question given below is one of them)

16. Let p_i be the probability that a randomly chosen point has i many friends, $i = 0,1,2,3,4$. Let X be a random variable such that for $i = 0,1,2,3,4$, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is

Ans. (24.00)

Sol.



P_i = Probability that randomly selected points has friends

$P_0 = 0$ (0 friends)

$P_1 = 0$ (exactly 1 friends)

$P_2 = \frac{{}^{4}C_1}{{}^{49}C_1} = \frac{4}{9}$ (exactly 2 friends)

$P_3 = \frac{{}^{20}C_1}{{}^{49}C_1} = \frac{20}{49}$ (exactly 3 friends)

$P_4 = \frac{{}^{25}C_1}{{}^{49}C_1} = \frac{25}{49}$ (exactly 4 friends)

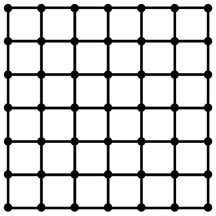
x	0	1	2	3	4
$P(x)$	0	0	$\frac{4}{49}$	$\frac{20}{49}$	$\frac{25}{49}$

Mean = E(x) = $\sum x_i P_i = 0 + 0 + \frac{8}{49} + \frac{60}{49} + \frac{100}{49} = \frac{168}{49}$

$7(E(x)) = \frac{168}{49} \times 7 = 24$

PARAGRAPH "II"

Consider the 6 × 6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



(There are two questions based on PARAGRAPH "II", the question given below is one of them)

17. Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots, A_{49}$. Let p be the probability that they are friends. Then the value of $7p$ is

Ans. (0.50)

Sol. Total number of ways of selecting 2 persons = ${}^{49}C_2$

Number of ways in which 2 friends are selected = 6 × 7 × 2 = 84

$7P = \frac{84 \times 2}{49 \times 48} \times 7 = \frac{1}{2}$